WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 3rd Semester Supplementary Examination, 2021

PHSACOR05T-PHYSICS (CC5)

Time Allotted: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *two* from the rest

1. Answer any ***ten*** questions from the following: 2×10=20

(a) Give an example of a piecewise continuous function within an interval and draw its graph schematically.

(b) What is periodic function? If $f(x)=\alpha x$ a periodic function (x being a real variable) where α is purely imaginary?

(c) Show that, for Laguerre equation $xy''+(1-x)y'+ay=0$, there is an essential singularity at infinity.

(d) Using the generating function of Bessel function given by $G(x,\, t)=e^{\frac{x}{2}(t-\frac{1}{t})}$, prove the following recursion relation: $J_{n-1}(x)+J_{n+1}(x)=\frac{2n}{x}J_n(x)$.

(e) Using the generating function of Hermite Polynomials, $G(x,\, h)=e^{2hx-h^2}=\sum_{n=0}^{\infty}\frac{H_n(x)}{n!}h^n$, show that $H_n(-x)=(-1)^n H_n(x)$.

(f) Find the value of $\Gamma(\frac{1}{2})$.

(g) Derive the relation $\Gamma(n+1)=n\Gamma(n)$.

(h) Find the Legendre transformation of the function $f(x)=e^x$.

(i) What are holonomic constraints? Give one example.

(j) Two point masses are connected by a massless rigid rod. Find the degrees of freedom of the system.

(k) Calculate the value of Poisson bracket, $[q,\, p^2+q^2]$ where symbols are bearing usual meaning.

(l) Consider the differential equation, $x^2(x-1)y''+4xy'+3y=0$

Find the singular points of this equation and their type of singularities.

(m) Show that the general solution of the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ is of the form $y = f_1(x-ct) + f_2(x+ct)$.

(n) Distinguish between an ordinary point and a singular point in connection with second order linear ordinary differential equations.

2. (a) Given $f(x) = \begin{cases} -1 & -\pi < x < 0 \\ +1 & 0 < x < \pi \end{cases}$ 3

Expand it in an appropriate Fourier series of period 2π.

(b) A Lagrangian is given in the form $L = \frac{1}{2}\alpha\dot{q}^2 - \frac{1}{2}\beta q^2$, where α and β are constants: 1+2

(i) Obtain the Lagrange's equation of motion.

(ii) Find the Hamiltonian of the system.

(c) Show that for the following equation, 2+2

$$xy'' + (1-x)y' + 4y = 0$$

about the point $x = 0$, the only possible solution of the indicial equation is 0. Find the recursion relations among coefficients appearing in the Frobenius series.

3. (a) Let $F(x)$ have a Fourier series expansion, 3

$$F(x) = \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad ; \quad (a_n\text{'s and } b_n\text{'s are real constants})$$

Then prove that $\langle F^2(x) \rangle \equiv \frac{1}{2\pi}\int_{-\pi}^{\pi} F^2(x)\,dx = \sum_{n=1}^{\infty} \frac{a_n^2 + b_n^2}{2}$.

(b) Given $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$, where $\vec{r}_1$ and $\vec{r}_2$ are position vectors of two points. Expand $\frac{1}{|\vec{r}_{12}|}$ in terms of Legendre polynomial. How do you interpret this result? 3+1

(c) Show that for any dynamical variable u, 3

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + [u, H],$$

where $[u, H]$ stands for Poisson bracket between u and H. Hence prove that Hamiltonian itself is a constant of motion, when it has no explicit dependence on time.

4. (a) Write down the generating function for Legendre polynomials. Hence show that $P_n(-x) = (-1)^n P_n(x)$. 1+2

(b) Define beta function $\mathcal{B}(m, n)$ and gamma function $\Gamma(n)$. Show that, $\mathcal{B}(m, n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. 2+2

(c) Using Euler-Lagrange equation, show that the shortest path on a plane connecting points is a straight line. 3

5\. (a) Given, $\dfrac{d}{dx}[x^{\nu} J_{\nu}(x)] = x^{\nu} J_{\nu-1}(x)$, and, $\dfrac{d}{dx}[x^{-\nu} J_{\nu}(x)] = -x^{-\nu} J_{\nu+1}(x)$, 3

show that, $J_{\nu-1}(x) + J_{\nu+1}(x) = \dfrac{2\nu}{x} J_{\nu}(x)$.

(b) Consider a single particle of mass *m* moving on a plane under a conservative force field *F*. Construct the Lagrangian in Cartesian co-ordinates and hence show that the Lagrange's equations for the particle are same as those coming from Newton's Second Law. 1+2

(c) A string of length *l* is stretched tightly and its ends are kept fixed. Find the general solution using separation of variables. 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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